

Evgeny Abakumov
Chui's conjecture in Bergman spaces

We discuss the extremal problem posed by Chui in 1971, along with several related optimization and approximation problems concerning sums of the Cauchy kernels.

Siskakis Aristomenis
Composition semigroups and related operators

Semigroups $(\phi_t)_{t \geq 0}$ of analytic self-maps of the unit disc \mathbb{D} induce composition operator semigroups and weighted versions,

$$T_t(f)(z) = f(\phi_t(z)), \quad S_t(f)(z) = m_t(z)f(\phi_t(z))$$

(with (m_t) a co-cycle for (ϕ_t)) on spaces of analytic functions on \mathbb{D} . We will review some selected results and resulting operators.

Olivia Constantin
Some flow characteristics of two-dimensional travelling water waves via Complex Analysis

In this talk we investigate certain flow quantities for the two most commonly observed types of two-dimensional travelling water waves, i.e. solitary waves and periodic travelling waves. Our methods are based on several classical convexity theorems due to Hardy et al..

Alberto Dayan
Cyclicity of singular inner functions on Besov and coefficient spaces

The seminal work of Beurling says that no singular inner function can be cyclic for the Hardy space on the unit disc. On the other hand, Korenblum showed that some singular inner functions are cyclic in Bergman-type spaces. In this talk, we show the existence of singular inner functions that are cyclic in Besov-type spaces. As a corollary, we will see how such singular inner functions are cyclic also for the space of those analytic functions with p -summable Taylor coefficient, for $p > 2$. Our condition relies only on the second modulus of continuity of the underlying singular measure, and hence is far more treatable than the one provided by Anderson, Fernández and Shields in the setting of the small Bloch space. Time permitting, we will also discuss whether such cyclic singular inner function are multipliers of the Besov and the coefficient spaces that we consider. This talk is based on a joint work with Daniel Seco.

Michael Jury
Determinants of Random Unitary Pencils

By a *random unitary pencil* we mean an expression of the form

$$L_X(U) := I_k \otimes I_d + \sum_{j=1}^g X_j \otimes U_j$$

where $X = (X_1, \dots, X_g)$ is a fixed g -tuple of $k \times k$ matrices, and we consider the U_j to be unitary matrices sampled independently with respect to Haar measure on the $d \times d$ unitary group $\mathcal{U}(d)$. Motivated by the rich theory that exists in the case of a single variable ($g = 1$) we describe a conjecture in the multivariate case ($g > 1$), about the limiting behavior of the covariances $\mathbb{E}[\det(L_X(U))\overline{\det(L_Y(U))}]$ for contractive g -tuples X and Y , as $d \rightarrow \infty$. The conjecture is true at least in the case of upper triangular X and Y , which leads to an interesting (but still poorly understood) connection between random matrix theory and the Drury-Arveson space. This is joint work with George Roman.

David Kalaj

Hopf Conjecture for Gaussian Curvature of Minimal Graphs

We address a refined form of the Gaussian curvature conjecture for minimal graphs, motivated by classical questions posed by Heinz and Hopf. Specifically, we consider minimal graphs defined over the unit disk and establish a sharp inequality for the Gaussian curvature at the point above the origin. Our approach reduces the problem to the analysis of Scherk-type minimal surfaces spanning bicentric quadrilaterals inscribed in the disk. Utilizing complex analytic techniques and the conformal harmonic parametrization of minimal surfaces, we derive precise curvature estimates that resolve the conjecture. This work builds on and completes prior efforts to understand the interplay between boundary geometry and intrinsic curvature in minimal surface theory.

Adem Limani

Summable analogues to the Ivashev-Musatov Theorem

We discuss summable versions of the classical Ivashev-Musatov Theorem in harmonic analysis, and a threshold phenomenon of Katznelson-type.

Bartosz Malman

Szegő's Theorem with a twist

Soon after I started my doctoral studies under Alexandru, he suggested for me to work on an approximation problem in spaces $\mathcal{H}(b)$. I had fun, I hope he did too, and we proved some stuff. Once, from behind his desk, he asked me if I could tell when $\mathcal{H}(b) \cap C^\infty$ was a large set. Over the years, I collaborated with Adem Limani and Linus Bergqvist on various aspects of this problem which turned out to be related to works of Khrushchev, Korenblum, Thomson, Volberg and others. In the talk, I want to report back to Alexandru what I know.

The basic problem involved needs not mention spaces $\mathcal{H}(b)$. Given a measure $w|dz|$ on the circle, we know from the work of Szegő that the analytic polynomials will approximate any function in $L^2(w)$ if and only if $\log w$ has a divergent integral. If the polynomial approximating sequence is to have additional properties, we need to ask $\log w$ to behave worse. One can prove theorems of the following type: if the integral of $\log w$ diverges over all sets in some class, then the approximating sequences can be chosen to behave especially nice, in one way or another. In my talk, I will phrase these results in the context of $P^t(\mu)$ -spaces, one topic of many to which Alexandru greatly contributed.

John McCarthy
The amazing Kosiński set

$$\mathcal{K} = \{(x, y, z) \in \mathbb{D}^3 : x + y + z = xy + yz + xz\}.$$

It originally appeared as the uniqueness set for a 3 point extremal Pick problem on the tridisk [L. Kosiński, Three-point Nevanlinna-Pick problem in the polydisc, Proc. Lond. Math. Soc. (3) **111** (2015), no. 4, 887–910.]

The Carathéodory metric on any bounded subset X of \mathbb{C}^n is defined by

$$d_X(\lambda, \mu) = \sup\{\rho(\phi(\lambda), \phi(\mu)) : \phi \in O(X, \mathbb{D})\},$$

where ρ is the hyperbolic metric on \mathbb{D} . If X is contained in some domain Ω , then a priori $d_X \geq d_\Omega$, since it is easier for a function to be a holomorphic map from X to \mathbb{D} than to extend to a holomorphic map from all of Ω to \mathbb{D} . We shall say that X is a Carathéodory set in Ω if $d_X(\lambda, \mu) = d_\Omega(\lambda, \mu)$ for all pairs λ, μ in X . This is a strong restriction. For example, if Ω is the ball, the only Carathéodory sets are retracts.

Kosiński and W. Zwonek proved that \mathcal{K} is a Carathéodory set for \mathbb{D}^3 [L. Kosiński and W. Zwonek, Extension property and universal sets, Canad. J. Math. **73** (2021), no. 3, 717–736.]

It turns out that every Carathéodory set in the polydisk is built out of copies of \mathcal{K} and \mathbb{D} . We shall describe how this happens.

This is ongoing joint work with L. Kosiński.

José Ángel Peláez

Composition of analytic paraproducts on classical spaces of analytic functions.

Let $\mathcal{H}(\mathbb{D})$ denote the space of analytic functions on the unit disc \mathbb{D} of the complex plane. For a fixed analytic function g on the unit disc \mathbb{D} , we consider the analytic paraproducts induced by g , which are defined by $T_g f(z) = \int_0^z f(\zeta)g'(\zeta) d\zeta$, $S_g f(z) = \int_0^z f'(\zeta)g(\zeta) d\zeta$, and $M_g f(z) = f(z)g(z)$. The boundedness of these operators on various spaces of analytic functions on \mathbb{D} is well understood. However, the boundedness of compositions of two of (more) of these operators had not been considered prior to the papers [1, 2] where this question was studied on the classical Hardy and Bergman spaces. In this talk we will recall some results obtained in these papers and we will present some recent results on the boundedness of compositions of analytic paraproducts on large Bergman spaces [3].

These results are part of several joint works together with A. Aleman, Carmé Cascante, Joan Fàbrega and Daniel Pascuas.

- [1] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Composition of analytic paraproducts*, J. Math. Pures Appl. (9) **158** (2022), 293–319.
- [2] A. Aleman, C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Words of analytic paraproducts on Hardy and weighted Bergman spaces*, J. Math. Pures Appl. (9) **188** (2024), 179–214.
- [3] C. Cascante, J. Fàbrega, D. Pascuas, and J. A. Peláez, *Words of analytic paraproducts on Bergman spaces induced by smooth rapidly decreasing weights*, preprint (submitted) <https://arxiv.org/abs/2504.09105>

Marco Pélolo

(Ir-)regularity of the Bergman projection on domains in \mathbb{C}^n

In this talk I will illustrate some of the problems that arise in several complex variables while studying the (ir-)regularity of the Bergman projections. Main goal of this talk is to illustrate some of the differences between the one-dimensional case and the case of several variables. I will discuss the case of worm domains in \mathbb{C}^2 , both in the Lebesgue and Sobolev spaces, in connection with other questions the underlying domain. I am going to present a new example of domain in \mathbb{C}^n , $n \geq 3$. Finally, I will discuss the case of homogeneous Siegel domains of Type II, where another phenomenon not-yet-observed in one complex dimension arises.

Karl-Mikael Perfekt

Hankel operators on Paley-Wiener spaces of convex domains

We consider Schatten classes for Hankel operators $H_\varphi : \mathbf{PW}(\Omega) \rightarrow \mathbf{PW}(\Omega)$,

$$\widehat{H_\varphi f}(x) = \int_{\Omega} \widehat{\varphi}(x+y) \widehat{f}(y) dy, \quad x \in \Omega,$$

on the Paley–Wiener space of a convex domain $\Omega \subset \mathbb{R}^n$,

$$\mathbf{PW}(\Omega) = \{f \in L^2(\mathbb{R}^n) : \text{supp}(\widehat{f}) \subset \overline{\Omega}\}.$$

For *admissible* domains, we prove that $H_\varphi \in S^p$ if and only if φ belongs to a certain Besov space of Paley–Wiener type, for $1 \leq p \leq 2$. For smooth domains $\Omega \subset \mathbb{R}^2$ with positive curvature, this result extends to $1 \leq p < 4$, and for simple polytopes in \mathbb{R}^n to the full range $1 \leq p < \infty$. The construction of the corresponding Besov spaces is quite technical, so the talk will focus on a selection of aspects.

Based on joint work with Konstantinos Bampouras (NTNU).

Alexei Poltoratski

Convergence of spectral and scattering data

This talk is devoted to applications of complex analysis to spectral problems for canonical Hamiltonian systems and scattering for Dirac systems of differential equations. I will discuss convergence results in the settings of inverse spectral problems and non-linear Fourier transform. The talk is partially based on joint work with Nikolai Makarov and Ashley Zhang.

Stefan Richter

Von Neumann’s inequality for row contractive matrix tuples

Drury’s von Neumann inequality says that if $d \in \mathbb{N}$ and if $T = (T_1, \dots, T_d)$ is a row contractive d -tuple of commuting Hilbert space operators, then $\|p(T)\| \leq \|p\|_{\text{Mult}(H_d^2)}$ for every polynomial p . Here $\|p\|_{\text{Mult}(H_d^2)}$ denotes the multiplier norm on the Drury-Arveson space H_d^2 . Furthermore, for $d > 1$ one cannot get an inequality of the type $\|p(T)\| \leq C\|p\|_\infty$, where $\|p\|_\infty = \sup_{|z| \leq 1} |p(z)|$.

We prove that for each $n \in \mathbb{N}$ there exists a constant C_n such that for all $d \in \mathbb{N}$ and for every commuting row contraction $T = (T_1, \dots, T_d)$ that consists of $n \times n$ matrices we have

$$\|p(T)\| \leq C_n \|p\|_\infty.$$

for every polynomial p . This is joint work with Michael Hartz and Orr Shalit.

Bill Ross
Aspects of the Hardy Operator

In this joint work with Eva Gallardo and Jonathan Partington, we study the classical Hardy averaging operators

$$(Hf)(x) = \frac{1}{x} \int_0^x f(t) dt$$

on $L^2(0, 1)$ and $L^2(0, \infty)$. In particular, we study aspects of the cyclic vectors, invariant subspaces, and related frame vectors.

Kristian Seip
The Hörmander—Bernhardsson extremal function

We characterize the function φ of minimal L^1 norm among all functions f of exponential type at most π for which $f(0) = 1$. This function, studied by Hörmander and Bernhardsson in 1993, has only real zeros $\pm\tau_n$, $n = 1, 2, \dots$. Starting from the fact that $n + \frac{1}{2} - \tau_n$ is an ℓ^2 sequence, established in an earlier paper of ours, we identify φ in the following way. We factor $\varphi(z)$ as $\Phi(z)\Phi(-z)$, where $\Phi(z) = \prod_{n=1}^{\infty} (1 + (-1)^n \frac{z}{\tau_n})$ and show that Φ satisfies a certain second order linear differential equation along with a functional equation either of which characterizes Φ . We use these facts to establish an odd power series expansion of $n + \frac{1}{2} - \tau_n$ in terms of $(n + \frac{1}{2})^{-1}$ and a power series expansion of the Fourier transform of φ , as suggested by the numerical work of Hörmander and Bernhardsson. The dual characterization of Φ arises from a commutation relation that holds more generally for a two-parameter family of differential operators, a fact that is used to perform high precision numerical computations. Joint work with Andriy Bondarenko, Joaquim Ortega-Cerdà, and Danylo Radchenko.

Alan Sola
Clark measures for rational inner functions

Reporting on joint work with J.T. Anderson, L. Bergqvist, K. Bickel, and J.A. Cima, I will present detailed results concerning Clark measures associated with general two-variable rational inner functions (RIFs) on the bidisk. Our results give precise descriptions of support sets and weights for such Clark measures in terms of level sets and partial derivatives of the associated RIF, and can be used to answer questions about mapping properties of Clark embeddings in the two-variable setting.

Dan Timotin
De Branges-Rovnyak spaces with rational reproducing kernels

De Branges-Rovnyak spaces are Hilbert spaces of analytic functions in the unit disk that have attracted much interest in the last years. In the general case its elements are not described explicitly, but if the reproducing kernel is rational one can give more concrete results. We will present a Smirnov type result as well as a corona theorem for multipliers on these spaces. The proof of the latter will involve a surprising estimate on the solutions of the classical Bézout equation, that we will discuss more at length.

This is joint work with Andreas Hartmann, Emmanuel Fricain, and William Ross.

Aron Wennman
An equivariant Weierstrass theorem

Consider the map Z from the space \mathcal{E} of non-constant entire functions to the space \mathcal{D} of discrete multi-sets without finite accumulation points (positive *divisors*), given by assigning to an entire function its zero set. The classical Weierstrass theorem states that the map Z is surjective. Hence it admits a right-inverse W , which assigns to each divisor Λ an entire function f with zero set $Z(f) = \Lambda$.

In the standard topologies, the zero set map Z is continuous. Moreover, the complex plane acts continuously by translation on both spaces \mathcal{E} and \mathcal{D} , and Z is *equivariant*, i.e., it commutes with translation. A natural question is whether one can construct an equivariant right-inverse W .

In ongoing joint work with Konstantin Slutsky and Mikhail Sodin, we find that, excluding periodic zero sets, there exists a Borel equivariant right inverse W of Z , but not a continuous one. Similar phenomena occur also in PDE, for equivariant solutions to Laplace and $\bar{\partial}$ -equations.

Artur Nicolau
Analytic maps of the disc with bounded Möbius distortion

The Möbius distortion of an analytic self-map of the disc measures how much it deviates from an automorphism. We will focus on maps with bounded Möbius distortion and discuss descriptions of these maps given in different terms: the existence of good hyperbolic geodesics, or the location of their zeros or the properties of their Aleksandrov-Clark measures. Joint work in progress with Oleg Ivrii.

Annemarie Luger
Quasi-Herglotz functions

With two Herglotz-Nevalinna functions f and g (i.e., analytic functions that map the complex upper halfplane \mathbb{C}^+ into itself) also their sum $f + g$ and positive scalar multiples $\lambda \cdot g$ (for $\lambda > 0$) obviously belong to the same class. But what can be said about arbitrary (complex) linear combinations?

This seemingly little extension leads to surprisingly interesting features. We will give an analytic characterization of these functions as well as discuss different properties and applications.

Parts of this talk are based in works with Christian Emmel and Mitja Nedic.